MATH 54 - HINTS TO HOMEWORK 1

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Here are a couple of hints to Homework 1! Enjoy :)

1. Section 1.1: Systems of linear equations

1.1.11, 1.1.13. All you have to do are row-reductions until it is easier to see whether the equation has a solution or not. In particular, if one of the rows is of the form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & b \end{bmatrix}$$

then the system has no solution!

1.1.23.

(a) T
(b) F (5 rows, 6 columns)
(c) T
(d) T

1.1.24.

(a) T (that's the point of EROS)
(b) Very F
(c) F (no sol)
(d) T

1.1.29. Just say how to obtain the second matrix from the first. For the reverse statement, how do you obtain the first matrix from the second?

Section 1.2: Row Reduction and Echelon Forms

1.2.7, 1.2.11, 1.2.13. Use Row-Reduction! I would strongly suggest for you to use the **Reduced** row-echelon form because that makes it easier to solve your system! **Be careful** about how many rows you eliminate in 1.2.11, don't eliminate all the rows, just two of them! In this case, there are 2 free variables!

1.2.21.

(a) F (the RREF is unique)
(b) F? (it also applies to coefficient matrices. Here it depends how you argue it)
(c) T
(d) T
(e) F (it just means one variable is 0)

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1.2.23, 1.2.24. In those 2 problems, the following fact will help you solve the problem:

Fact: A system is consistent if and only if in the row echelon form of the augmented matrix there is no row of the form

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & b \end{bmatrix}$$

Where $b \neq 0$.

For 1.2.23, it's yes because there is no row of the above form, for 1.2.24 the answer is no because there is a row of the above form!

Section 1.3: Vector equations

1.3.5, 1.3.9. For 1.3.5, your answer should be of the same form as 1.3.9, and vice-versa

1.3.11. In other words, does the system $x_1\mathbf{a_1} + x_2\mathbf{a_2} + x_3\mathbf{a_3} = \mathbf{b}$ (where the unknowns are x_1, x_2, x_3) have a nonzero solution?

1.3.13. This is the same as 1.3.11, where $\mathbf{a_1}$ is the first column of A, $\mathbf{a_2}$ is the second column etc. In other words just form the augmented matrix $[A \mid \mathbf{b}\ ]$ and row-reduce!

1.3.15. Span $\{\mathbf{v_1}, \mathbf{v_2}\}$ is just the set of linear combinations of $\mathbf{v_1}$ and $\mathbf{v_2}$. So for example, the following vectors are in that Span: $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_1} + \mathbf{v_2}, 2\mathbf{v_1}, 3\mathbf{v_1} - 2\mathbf{v_2}$, etc.

1.3.23.

(a) T
(b) F
(c) T
(d) T
(e) F (it could be a line, for example if $\mathbf{u} = \mathbf{v}$, it could even be the $\mathbf{0}$ vector)

1.3.24.

(a) T
(b) T
(c) F
(d) T (yes, but it contains other things too, like $\mathbf{v}$, the line through $\mathbf{v}$ and the origin, etc.)
(e) T

Section 1.4: The matrix equation $Ax = b$

1.4.3. Look at page 43

1.4.7. In other words, just erase x_1, x_2, x_3 and put the resulting 3 column vectors in a matrix

1.4.11. Row-reduce!

1.4.23.

(a) F (*matrix* equation)
(b) T
(c) F (not quite, there *could* be a pivot in the last column, such as $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$)
(d) T
(e) T
(f) T (if A had a pivot position in each row, then there can't be a row of the form $\begin{bmatrix} 0 & 0 & 0 & b \end{bmatrix}$ in the augmented matrix corresponding to the equation $A\mathbf{x} = \mathbf{b}$, compare with 1.2.23)

1.4.24.

(a) T
(b) T
(c) T
(d) T
(e) F
(f) T (otherwise the equation would be consistent for all $\mathbf{b}$)

SECTION 1.5: SOLUTION SETS OF LINEAR SYSTEMS

Note: You can easily do the problems in this section without reading it, it's basically a review of how to solve linear equations!

1.5.9, 1.5.1. Row-reduce!

1.5.13. Add the trivial equation $x_3 = x_3$ to your list of equations, and then you find that the solution set is of the form: $\mathbf{x} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$

1.5.23.

(a) **T** ($\mathbf{x} = \mathbf{0}$ is always a solution)
(b) **T?** (the question is a bit weirdly phrased)
(c) **F** (no, it means that $\mathbf{x} = \mathbf{0}$ solved the equation)
(d) **F** ($\mathbf{p}$ and $\mathbf{v}$ should be flipped)
(e) **T**

1.5.24.

(a) **F** (nontrivial means not all entries are 0. For a counterexample, let A be the zero matrix!)
(b) **T**
(c) **T!** (this got me too! But if $\mathbf{x} = \mathbf{0}$ is a solution, then $\mathbf{b} = \mathbf{0}$ and the equation is homogeneous)
(d) **F** (no, draw a picture, or see page 55)
(e) **T** (see page 55)